

Friday 9

The Electro-Mechanical Analogy

Analogies occur throughout physics. There is a very close analogy between mechanical and electrical motion. Many quantities that we use to describe mechanical motion have an analog in electrical motion.

Mechanical Quantity	Symbol	Electrical quantity	Symbol
mass	m	inductance	L
spring constant	k	capacitance	C
force	F	voltage	V
velocity	v	current	I
impedance= p/v	Z	resistance=V/I	R

Flow

Sound is not a steady flow of air. However, if you have a steady pressure difference you can have steady flow of air down a pipe. You determine the total force on the air by multiplying the pressure by the area of the pipe. The relationship between the flow velocity and the pressure difference is an impedance which is due to the viscosity of air. For an electrical circuit, voltage is the equivalent of force. If there is a voltage difference across a device then current will be forced to flow. The relationship between force and motion for electricity is called *Ohm's law of electricity* (It is from the same professor Ohm who has the law of hearing.)

$$p = v \cdot Z$$

$$V = I \cdot R$$

Voltage V is measured in the unit of *Volts*. Electrical flow I (called current) is measured in *Amperes (or Amps)*. The electrical impedance R is called resistance and is measured in the unit of *Ohms*. Ohm's law applies both for steady flow and for an oscillating flow of electricity. If you know that a certain device draws 20 Amps when you plug it into the wall plug, then you can figure out how much resistance it has. The wall plug provides a Voltage of 120V so you know that

$$120(\text{Volts}) = R(\text{Ohms}) \cdot 20(\text{Amps})$$

So the resistance must be 6 Ohms. One Amp times one Ohm equals one Volt.

Power

For sound in tubes we found that the total power was equal to the Force times velocity. Force equals pressure times the area of the tube. Power= $p \cdot A \cdot v$. For electricity it is equal to the voltage multiplied by the current. Power= $V \cdot I$. Using this equation together with Ohm's law we obtain equations for the power analogous to what we had for sound power:

$$P = \frac{V^2}{R}$$

$$P = I^2 R$$

Resonance.

Using this analogy it is possible to make simple harmonic electrical oscillators. The two ingredients of inertia and restoring force can also be accomplished electrically.

Inertia

An inductance is a coil of wire. When electricity flows through a coil it makes a magnetic field. You can't change the current in a coil without exerting a force (voltage). This is analogous to Newton's second law that you can't change the velocity of a mass without exerting a force.

$m \cdot v = \text{force multiplied by time.}$

$L \cdot I = \text{voltage multiplied by time.}$

Restoring force

Electricity is a flow of electrons. Electrons repel each other, so if they are compressed into a certain place there is a force that pushes back. A capacitor is a device that electrons can be forced into by a voltage. This is analogous to the restoring force of a spring constant. The only difference is that a larger the capacitance is like a weaker spring. Strictly speaking the spring constant k is analogous to the inverse of capacitance $1/C$.

$$F = kx$$

$$V = \frac{1}{C}Q$$

In this way, connecting an inductance to a capacitance is like connecting a mass to a spring.

Impedance matching.

In wave systems you get the maximum transfer of power (minimum reflection) if you match the impedance between two media. In electrical circuits it is the same thing. Every electrical power source (for example an audio power amplifier) has an internal impedance. It will deliver maximum power (for example to a loudspeaker) if the impedance of the loudspeaker is equal to the amplifier impedance. We will do a simple example of this as a demonstration.

In electrical jargon, when you connect an electrical power source to something and deliver power to it, that something is called a *load*. When the load impedance is equal to the source impedance, then maximum power is transferred to the load.

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Electrical impedance matching.

Name _____

Section _____

For each adjustment of the circuit we will measure voltage and current. From that you can calculate the resistance using ohms law ($R=V/I$) and the power using the formula $P=VI$. The circuit has an internal resistance of 500 ohms. Can you tell this from your results?

Voltage (volts)	current (milliamps)	resistance (ohms)	power (milliwatts)
6	.6	10,000	3.6
5.5	1.6	3440	8.8
5	2.5	2000	12.5
4.5	3.5	1300	15.75
4	4.5	900	18
3.5	5.5	640	19.25
3	6.33	474	19
2.5	7.4	338	18.5
2	8.25	242	16.5
1.5	9.3	161	14

